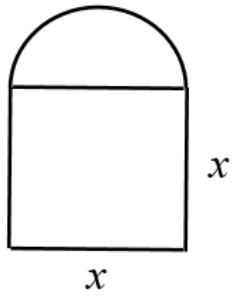
Exercise 105

A window has the shape of a square surmounted by a semicircle. The base of the window is measured as having width 60 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum error possible in computing the area of the window.

Solution

Draw a schematic of the window.



The area is

$$\begin{aligned} A &= x^2 + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 \\ &= x^2 + \frac{1}{2}\pi\left(\frac{x^2}{4}\right) \\ &= x^2 + \frac{\pi x^2}{8} \\ &= \frac{1}{8}(8+\pi)x^2. \end{aligned}$$

Take the derivative.

$$\begin{aligned} \frac{dA}{dx} &= \frac{d}{dx}\left[\frac{1}{8}(8+\pi)x^2\right] \\ &= \frac{1}{8}(8+\pi)(2x) \\ &= \frac{1}{4}(8+\pi)x \end{aligned}$$

As a result, the differential of area is

$$dA = \frac{1}{4}(8+\pi)x\,dx.$$

If $x = 60$ and $dx = 0.1$, then the maximum possible error in computing the area is

$$dA = \frac{1}{4}(8+\pi)(60)(0.1) = 12 + \frac{3\pi}{2} \approx 16.7124 \text{ cm}^2.$$